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An Outflow Boundary Condition for Aeroacoustic Computations

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ABSTRACT

We present a formulation of boundary condition for flows with small disturbances. We test our methodology in an axisymmetric jet flow calculation, using both the Navier-Stokes and Euler equations. Solutions in the far field are assumed to be oscillatory. If the oscillatory disturbances are small, the growth of the solution variables can be predicted by linear theory. We use the eigenfunctions of the linear theory explicitly in our formulation of the boundary conditions. This guarantees correct solutions at the boundary in the limit where the predictions of linear theory are valid.

Keywords: Nonreflecting boundary condition, Computational aeroacoustics, Jet flow computations

INTRODUCTION

Any attempt to directly compute the noise source from the flow field demands high accuracy of the numerical methods, including the treatment of the boundary conditions. Treatment of the outflow boundary for stable and accurate flow simulations has attracted considerable attention [see for example Engquist and Majda (1977), Bayliss and Turkel (1982), Scott and Hankey (1985), Hagstrom and Hariharan (1988), Roe (1989), Giles (1990), Hariharan and Hagstrom (1990), Thompson (1990), Tam and Webb (1993), Atkins and Casper (1994)]. One usually idealizes a physical problem to formulate the conditions at the boundary. The effectiveness of the boundary condition is dependent on the degree of validity of the idealized assumptions in the actual flow situation. Approaches based on linear analysis, especially variations of the characteristic methods, are widely used. Various investigators have derived essentially the same asymptotic pressure boundary conditions [See Hayder and Turkel(1994) for a discussion of various works on this boundary condition, an evaluation of its effectiveness, and comparisons with other boundary conditions]. Hayder and Turkel (1994) observed that the asymptotic pressure boundary condition gave reasonable results. They however recommended a small exit region beyond the region of interest. Their experiments indicated that Giles(1990) and characteristic boundary conditions with a larger exit layer also yields reasonable solutions. Because of the difference in the asymptotic forms of wave equations in two and three dimensions, this boundary condition is slightly different in three dimensions from two dimensions [see Hayder and Turkel (1994)]. Hariharan and Hagstrom (1990) formulated higher order forms of this boundary condition. As we stated earlier, a boundary condition will give satisfactory results if the assumptions used to derive the condition closely follow the actual flow situation.

In this paper we present a new approach to boundary treatment based on the linear stability theory. The governing equations of the fluid flow are nonlinear. However, if a mean flow is excited by a small disturbance, the linear theory can be used to predict its growth. Also, the eigenfunctions given by the linear theory describe the profiles of the disturbances after an initial adjustment region. This phenomenon motivates our present effort to find a boundary condition for a flow with small disturbances. We as-

sume the profiles of the disturbances at the outflow can be approximated by the eigenfunctions predicted by the linear theory. The particular eigenfunctions chosen would generally correspond to the most unstable modes. However, any eigenmodes could, in principle, be used. The latter may be relevant for forced problems, where the excited modes may be determined by the forcing. The boundary condition that is developed here should be accurate for cases where the linear theory accurately describes the disturbances at the boundary and where these are dominated by a single, known mode. It may not be appropriate when the nonlinearity in the flow is significant.

In Sections 2 and 3, we give, respectively, the governing equations for our test problem and the derivation of our new boundary condition. A discussion of the basic scheme for our test problem is given in Section 4 and we present our results in Section 5.

GOVERNING EQUATIONS

We compute the flow field of an axisymmetric jet to test our new boundary conditions. We solve the Navier-Stokes equations as given below

$$Q_t + F_x + G_r = S$$

where

$$Q = r\begin{pmatrix} \rho \\ \rho u \\ \rho v \\ E \end{pmatrix}$$

$$F = r\begin{pmatrix} \rho u \\ \rho u^2 - \tau_{xx} + p \\ \rho uv - \tau_{xr} \\ \rho uH - u\tau_{xx} - v\tau_{xr} - \kappa T_x \end{pmatrix}$$

$$G = r\begin{pmatrix} \rho v \\ \rho uv - \tau_{xr} \\ \rho v^2 - \tau_{rr} + p \\ \rho vH - u\tau_{xr} - v\tau_{rr} - \kappa T_r \end{pmatrix}$$

$$S = \begin{pmatrix} 0 \\ 0 \\ p - \tau_{\theta\theta} \\ 0 \end{pmatrix}$$

Q represents the solution variables, F and G are the fluxes in the x and r directions respectively, S is the source term that arises in the cylindrical polar coordinates, and τ_{ij} are the shear stresses.

DERIVATION OF THE BOUNDARY CONDITION

The governing equations for the fluid flows are the Navier Stokes equations. For high Reynold number flows, the viscous effect is small and one has to decide whether the boundary treatment should be based on the Euler equations or the Navier-Stokes equations. The difference between the two approaches is not just the type of boundary conditions but even the number of boundary conditions that need to be given. For inviscid subsonic flow, one boundary condition needs to be specified at outflow corresponding to an incoming acoustic wave. For supersonic flow no boundary condition is required. For viscous flow the equations are no longer hyperbolic but rather incompletely parabolic. For inflow, four conditions need to be specified (in two dimensions), while for outflow, three conditions need to be specified. In particular, the number of conditions does not depend on whether the local flow is subsonic or supersonic. Many codes use inviscid type boundary conditions. This is based on the assumption that the flow in the far field is essentially inviscid because of the high Reynolds number and the lack of physical boundary layers. Hayder and Turkel (1993) considered a framework for implementing the boundary conditions, where the formulation is of a characteristic type, but where viscous effects are also partially accounted for. We will use that framework for our present study. At subsonic outflow, we extrapolate three characteristic variables from the interior and impose one boundary condition. This is done by solving the following set of equations.

$$\begin{aligned} p_t - \rho c u_t &= R_1 \\ p_t + \rho c u_t &= R_2 \\ p_t - c^2 \rho_t &= R_3 \\ v_t &= R_4 \end{aligned} \tag{1}$$

where R_i is determined by which variables are specified and which are not. Whenever, the combination is not specified, R_i is just those spatial derivatives that come from the Navier-Stokes equations. Thus, R_i contains viscous contributions even though the basic format is based on inviscid characteristic theory. In implementing these differential equations we convert them to conservation variables ρ , m = ρ u , n = ρ v and E. Assuming an ideal gas we then have

$$p_t = (\gamma - 1)(E_t + \frac{u^2 + v^2}{2}\rho_t - u m_t - v n_t)$$

$$u_t = \frac{m_t}{\rho} - \frac{u\rho_t}{\rho}$$

$$v_t = \frac{n_t}{\rho} - \frac{v\rho_t}{\rho}$$

For subsonic outflow we calculate R_2, R_3, R_4 from the Navier-Stokes equations and set R_1 as prescribed by the given boundary condition. For supersonic flows, all the R_i at the outflow boundary can be calculated from the Navier-Stokes equations or else by extrapolation of all the characteristic variables from the interior.

In this work, we assume the solution variable Q at the outflow behaves as

$$Q(x,r,t) = \bar{Q}(x,r) + Q'(x,r,t)$$

where $\bar{Q}$ is the mean and Q' is the oscillatory part of the variable Q and

$$Q' = e^{\alpha_i x}[C_1\ cos(\alpha_r x - \omega t) + C_2\ sin(\alpha_r x - \omega t)]$$

Thus at outflow,

$$\begin{pmatrix} \rho' \\ u' \\ v' \\ p' \end{pmatrix} = \begin{pmatrix} \rho_r \\ u_r \\ v_r \\ p_r \end{pmatrix} A\ cos\omega t + \begin{pmatrix} \rho_i \\ u_i \\ v_i \\ p_i \end{pmatrix} B\ sin\omega t$$

where ω is the excitation frequency. Here, the vectors determining the structure of the disturbances are eigenfunctions of the linear stability equations. They are used explicitly in our boundary conditions. This form for the disturbances should hold if the solution is well-approximated by linear theory.

$$\begin{aligned} R_1 &= p_t - \rho c u_t \\ &= \omega\,(-A\,p_r \sin\omega t + B\,p_i \cos\omega t) \\ &\quad - \rho c\omega(-A\,u_r \sin\omega t + B\,u_i \cos\omega t) \end{aligned}$$

$$\begin{aligned} R_2 &= p_t + \rho c u_t \\ &= \omega\,(-A\,p_r \sin\omega t + B\,p_i \cos\omega t) \\ &\quad + \rho c\omega(-A\,u_r \sin\omega t + B\,u_i \cos\omega t) \end{aligned}$$

$$\begin{aligned} R_3 &= p_t - c^2\rho_t \\ &= \omega\,(-A\,p_r \sin\omega t + B\,p_i \cos\omega t) \\ &\quad - c^2\omega(-A\,\rho_r \sin\omega t + B\,\rho_i \cos\omega t) \end{aligned}$$

Let $\hat{A}$ = A ω sinωt and $\hat{B}$ = B ω cosωt. Then

$$\begin{pmatrix} -p_r - \rho c u_r & p_i + \rho c u_i \\ -p_r + c^2\rho_r & p_i - c^2\rho_i \end{pmatrix} \begin{pmatrix} \hat{A} \\ \hat{B} \end{pmatrix} = \begin{pmatrix} R_2 \\ R_3 \end{pmatrix}$$

or

$$\begin{pmatrix} \hat{A} \\ \hat{B} \end{pmatrix} = \frac{1}{\Delta} \begin{pmatrix} p_i - c^2\rho_i & -p_i - \rho c u_i \\ p_r - c^2\rho_r & -p_r - \rho c u_r \end{pmatrix} \begin{pmatrix} R_2 \\ R_3 \end{pmatrix}$$

$$\Delta = (c^2\rho_i - p_i)(\rho c u_r + p_r) + (p_i + \rho c u_i)(p_r - c^2\rho_r)$$

$$R_1 = \hat{A}(-p_r + \rho c u_r) + \hat{B}(p_i - \rho c u_i) \qquad (2)$$

Equation (2) is our new boundary condition and we use this value of R_1 in equation (1) for our numerical tests. We would like to point out that one can implement our new boundary condition [equation (2)] in other frameworks. For example, the framework presented in Tam and Webb(1993) uses the linearized Euler equations. One can implement our condition by replacing the condition corresponding to the incoming acoustic wave by equation (2). One may expect to see some differences with the same boundary condition is implemented in different ways.

We note that a similar construction could, in principle, be carried out at the inflow boundary. To do so, an eigenmode corresponding to a left-moving wave should be identified and its amplitude related to the outgoing characteristic variable, R_1. Finally, the incoming variables could be specified, again using the assumed form of the disturbance. We have not yet explored the feasibility of this approach.

BASIC SCHEME

We use a high order extension of the MacCormack Scheme due to Gottlieb and Turkel(1976). It has a predictor and a corrector stage and may be written as:

The predictor step with forward differences is

$$\bar{Q}_i = Q_i^n + \frac{\Delta t}{6\Delta x}\{7(F_{i+1}^n - F_i^n) - (F_{i+2}^n - F_{i+1}^n)\} + \Delta t S_i^n$$

The corrector step with backward differences is

$$Q_i^{n+1} = \frac{1}{2}[\bar{Q}_i + Q_i^n$$

$$+\frac{\Delta t}{6\Delta x}\{7(\bar{F}_i - \bar{F}_{i-1}) - (\bar{F}_{i-1} - \bar{F}_{i-2})\} + \Delta t\bar{S}_i]$$

This scheme is second order in time and becomes fourth-order accurate in the spatial derivatives when alternated with symmetric variants. We define L_1 as a one dimensional operator with a forward difference in the predictor and a backward difference in the corrector. Its symmetric variant L_2 uses a backward difference in the predictor and a forward difference in the corrector. For our computations, the sweeps are arranged as

$$Q^{n+1} = L_{1x}L_{1r}Q^n$$

$$Q^{n+2} = L_{2r}L_{2x}Q^{n+1}$$

Further description of our implementations can be found in Hayder et al.(1993) and Mankbadi et al.(1994).

RESULTS

We test our new boundary condition for an axisymmetric jet flow calculation. Details of such calculations can be found in Hayder et al. (1993) We note that the flow is

unstable, and hence provides a stiff test for any boundary condition. Here, the initial axial velocity is specified as

$$\bar{u}(r) = \frac{1}{2}[(1 + u_\infty) - (1 - u_\infty)tanh(4(r-1))]$$

and the corresponding temperature is given by the Busemann -Crocco integral of the energy equation:

$$\bar{T}(r) = T_0 + \frac{(\gamma-1)}{2}M^2(1-\bar{u})(\bar{u}-u_\infty).$$

Here $u_\infty = .25$ and the jet center temperature is assumed to be equal to the outer flow temperature, i.e., $T_0 = T_\infty$. The jet Mach number is M=1.5 and Reynolds number based on the jet radius is 364,000. We excite the inflow profile at location r and time t as

$$W(r,t) = \bar{W}(r) + \epsilon Re(W' e^{i\omega t})$$

where $W = (\rho, u, v, p)^T$, $\bar{W}$ is the mean and W' is the eigenfunction of the linear stability equations corresponding to the mean flow profile which has the most rapid growth rate. For our numerical tests we used $\omega = 1.08$ and $\epsilon = 10^{-6}$. Eigenfunctions (EF) are obtained by solving the linear stability equations and also using our flow code. In our implementation of the new boundary condition, we use for the eigenfunctions the average of those obtained from the linear stability calculations and those computed from our flow code, i.e.,

$$EF_{bc} = \frac{EF_{Linear\ Theory} + EF_{code}}{2}$$

We show contours of vorticity magnitude for Navier Stokes computations Figure 1. Similar computations with Euler equations are shown in Figure 2. We used three computational domains. They are 60, 50 and 40 radii long. All three computational domains are 5 radii wide in the transverse direction. The results at the outflow boundary of the shorter domains i.e., $x_l = 40$ and 50 compare well with the same quantities at the same locations in the long domains. The present boundary condition gave satisfactory results. Because of very high Reynolds number used in our computations, there is virtually no difference between our solutions of the Euler and the Navier Stokes equations.

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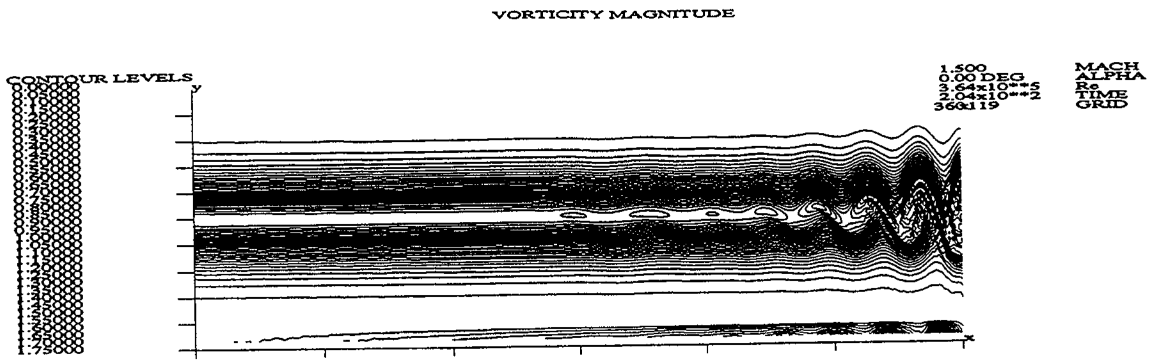

Figure 1a: Long Computational Domain Solution (xl=60)

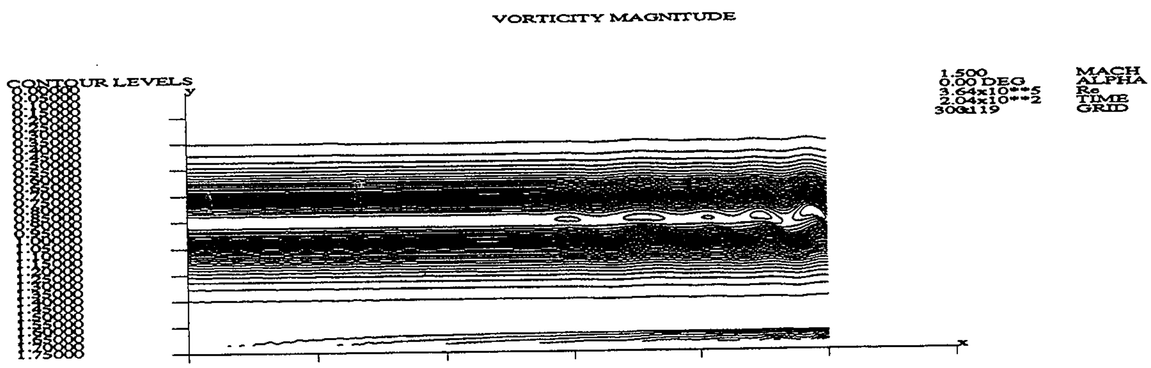

Figure 1b: Intermediate Computational Domain Solution (xl=50)

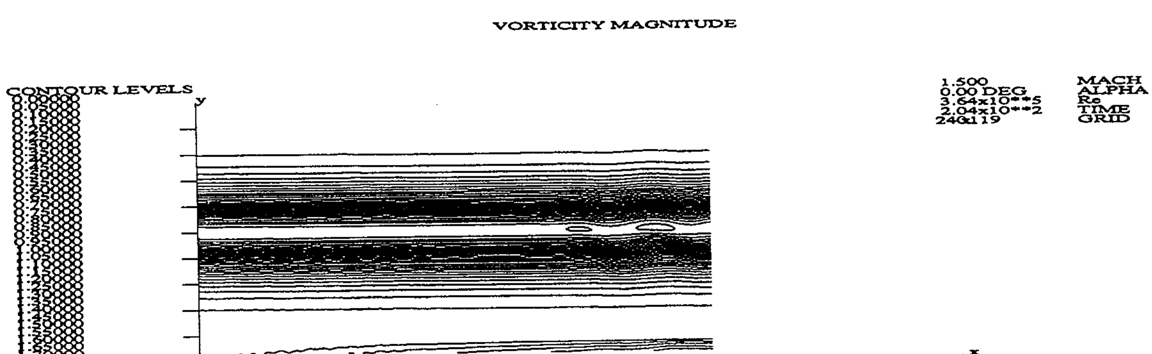

Figure 1c: Short Computational Domain Solution (xl=40)

Figure 1: Solutions of the Navier-Stokes equations

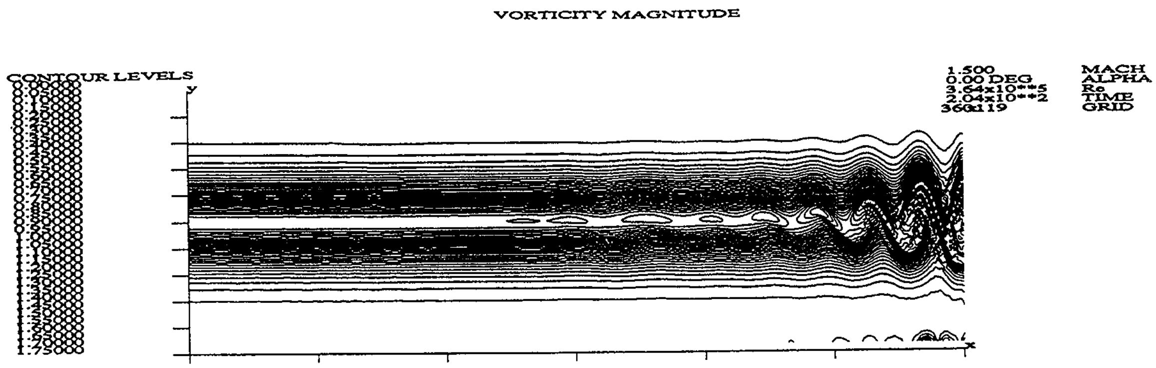

Figure 2a: Long Computational Domain Solution (xl=60)

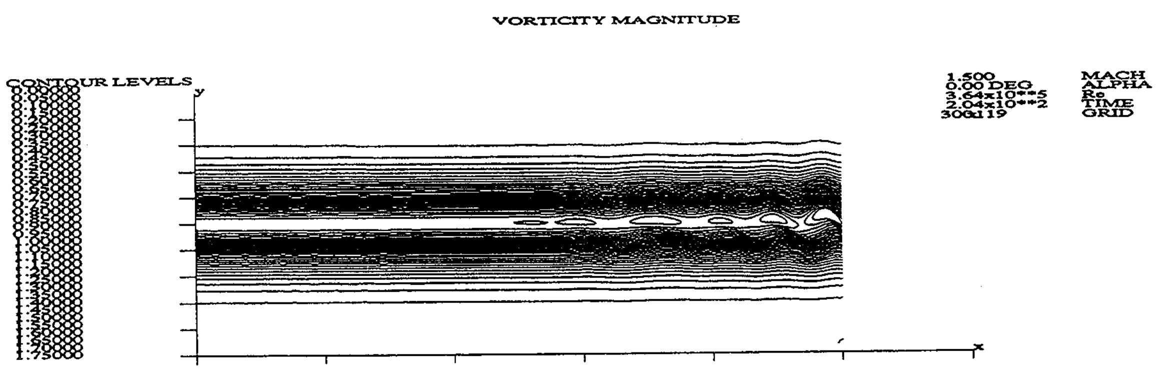

Figure 2b: Intermediate Computational Domain Solution (xl=50)

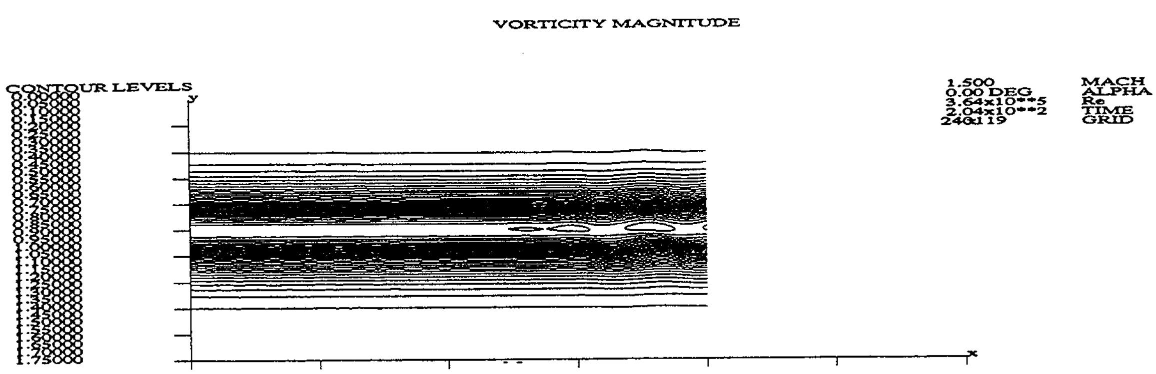

Figure 2c: Short Computational Domain Solution (xl=40)

Figure 2: Solutions of the Euler equations

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